$$\frac{df}{dR}=1-\Psi\frac{d^2\Psi}{dR^2}\Big/\left(\frac{d\Psi}{dR}\right)^2 \tag{A.11}$$

give (14).

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Microstrip Dispersion in a Wide-Frequency Range

EIKICHI YAMASHITA, KAZUHIKO ATSUKI, AND TETSUYA HIRAHATA

***Abstract*—The dispersion property of microstrip lines was measured in a frequency range from 2 to 50 GHz and compared with that estimated by an approximate formula in a previous paper.**

I. INTRODUCTION

The frequency dependence of wave velocity in a transmission line is called dispersion property, and is an important quantity when the line is used in a wide frequency range. The ratio of the propagation constant of the transmission line to that of free space β/β_0 is also used as a quantity to express the dispersion property. The dispersion property can be exactly analyzed by solving Maxwell's equations as a boundary value problem. The integral equation method [1] and the mode-matching method [2], among others, showed good agreement in the results of the analysis of microstrip dispersion. However, an approximate formula of the microstrip dispersion is needed in desk calculations and the CAD of microwave integrated circuits. Though a few approximate formulas have been reported in the past based on some physical considerations and experimental data for frequencies up to 12 GHz, [3]–[6], these empirical formulas have had narrow ranges of applicability. We also derived an approximate formula of microstrip dispersion [7] from the numerical result of the integral equation method [1].

Manuscript received September 5, 1980; revised December 24, 1980. This work was supported in part by a Grant in aid for Scientific Research on Microwave and Optical Planar Circuits.
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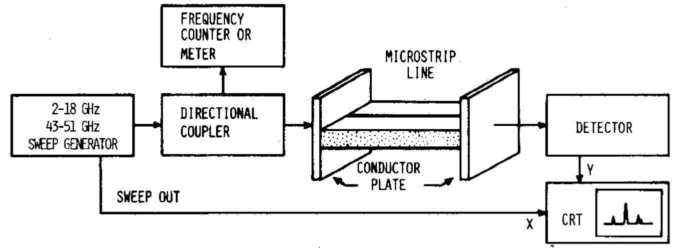

Fig. 1. Experimental setup for measuring the dispersion property of microstrip lines.

This paper describes the experimentally measured dispersion property of some microstrip lines in a wide-frequency range compared with that estimated by the above approximate formula.

II. APPROXIMATE DISPERSION FORMULA

The approximate formula of microstrip dispersion given in a previous paper [7] is

$$\frac{\beta}{\beta_0}=\frac{\sqrt{\epsilon^*}-\dfrac{\beta_{TEM}}{\beta_0}}{1+4F^{-1.5}}+\frac{\beta_{TEM}}{\beta_0} \tag{1}$$

where

$$F=\frac{4h\sqrt{\epsilon^*-1}}{\lambda_0}\left[0.5+\left\{1+2\log_{10}\left(1+\frac{w}{h}\right)\right\}^2\right] \tag{2}$$

β_{TEM} the propagation constant derived with the quasi-TEM wave approximation,
λ_0 wavelength in vacuum,
h the height of the substrate,
w the width of the strip conductor,
ϵ^* the dielectric constant of the substrate.

The applicable ranges of this formula are

$$2<\epsilon^*<16$$
$$0.06<w/h<16$$
$$0.1\ \text{GHz}<f<100\ \text{GHz}.$$

Though the lowest usable frequency is limited by 0.1 GHz, the propagation constant for frequencies less than 0.1 GHz has been already given as β_{TEM}.

III. EXPERIMENTAL RESULTS

The microstrip dispersion was measured with a resonance method. Fig. 1 shows an experimental setup for measuring the guide wavelength λ and the free space wavelength λ_0. The

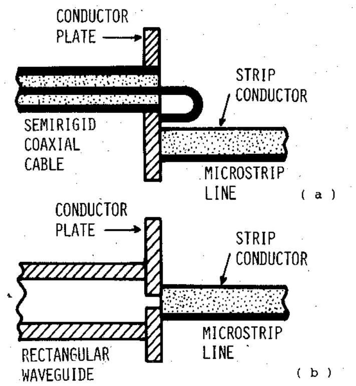

Fig. 2. Coupling structures between one end of the microstrip line resonator and (a) a coaxial cable (2–18 GHz), or (b) a waveguide (35 and 50 GHz).

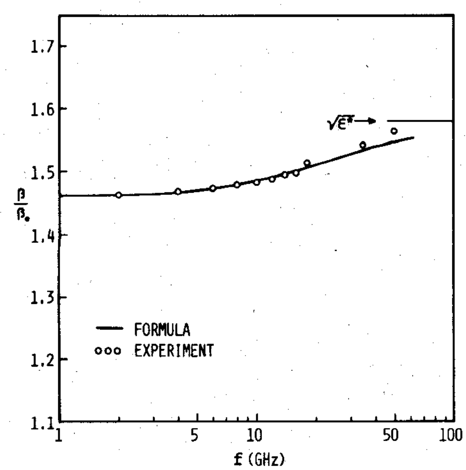

Fig. 3. The measured dispersion of a microstrip line with a Fluorglas substrate. $\epsilon^*=2.5$; $w/h=3.04$; $h=1.15$ mm.

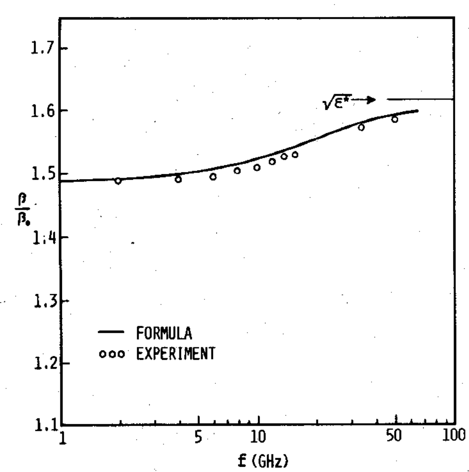

Fig. 4. The measured dispersion of a microstrip line with a Rexolite substrate. $\epsilon^*=2.62$; $w/h=2.82$; $h=1.57$ mm.

resonator is made of a microstrip line shorted at both ends by conductor plates. λ is measured as the resonator length divided by the number of standing waves between the conductor plates. λ_0 is the wave velocity divided by the frequency reading on the microwave counter (2–18 GHz) or wave meter (35 and 50 GHz). The ratio, λ_0/λ, equals to β/β_0. Fig. 2 illustrates two types of coupling structures between one end of the microwave resonator

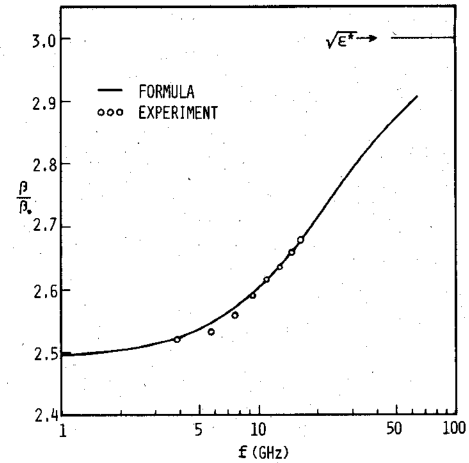

Fig. 5. The measured dispersion of a microstrip line with a Alumina substrate. $\epsilon^*=9.0$; $w/h=0.867$; $h=0.97$ mm.

and a coaxial cable (2–18 GHz) or a waveguide (35 and 50 GHz) in the experimental setup.

Figs. 3, 4, and 5 show the results of measurements by using the above experimental setup for three substrates: Fluorglas ($\epsilon^*=2.5$), Rexolite ($\epsilon^*=2.62$), and Alumina ($\epsilon^*=9.0$). Reasonable agreement between the formula and experimental results in these figures indicates the practicality of the approximate formula.

ACKNOWLEDGMENT

The authors thank Dr. Y. Suzuki for his helpful comments.

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Characteristic Impedances of Four-Conductor Transmission Line

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***Abstract*—A general formula for calculation of the characteristic impedance of four conductor transmission line in a rectangular shield is derived. A number of coupled and single strip transmission lines are considered by simplifying the general formula. Numerical results for a line in a square shield are presented graphically.**

Manuscript received September 3, 1980; revised January 8, 1981.
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0018-9480/81/0600-0611$00.75 ©1981 IEEE